PERIMETER, AREA & VOLUME

Rectangle

$P = 2l + 2w$

$A = lw$

Square

$P = 4s$

$A = s^2$

Triangle

$P = add\ all\ sides$

$A = \frac{1}{2}bh$

Parallelogram

$P = add\ all\ sides$

$A = bh$

Trapezoid

$P = add\ all\ sides$

$A = \frac{1}{2}(b_1 + b_2)h$

Circle

$C = \pi d = 2\pi r$

$A = \pi r^2$

Arc Length

$S = \theta r$ in radians

$S = \frac{\pi}{180}\theta r$ in degrees

Circle Sector Area

$A = \frac{\theta}{2}r^2$ in radians

$A = \frac{\theta}{360}\pi r^2$ in degrees

Rectangular solid

$S = 2lw + 2lh + 2wh$

$V = lwh$

Cube

$SA = 6s^2$

$V = s^3$

Cylinder

$SA = 2\pi r^2 + 2\pi rh$

$V = \pi r^2 h$

Cone

$SA = \pi rs + \pi r^2$

$V = \frac{1}{3}\pi r^2 h$

$A = \pi r\sqrt{r^2 + h^2}$

Sphere

$SA = 4\pi r^2$

$V = \frac{4}{3}\pi r^3$

$A = 4\pi r^2$

Rectangle Pyramid

$SA = lw + 2ls + 2ws$

$V = \frac{1}{3}lwh$

EXPONENT LAWS

$x^0 = 1$ if $x \neq 0$

$x^1 = x$

$x^{-n} = \frac{1}{x^n}$ if $x \neq 0$

$x^m . x^n = x^{m+n}$

$(x^m)^n = x^{m.n}$

$x^m \div x^n = \frac{x^m}{x^n} = x^{m-n}$ if $x \neq 0$

$(xy)^m = x^m y^m$

$\left(\frac{x}{y}\right)^n = \frac{x^n}{y^n}$ if $y \neq 0$

$x^{\frac{m}{n}} = \sqrt[n]{x^m}$ if ($a \geq 0$, $m \geq 0$, $n > 0$)

PROPERTIES OF LOGARITHMS

$y = \log_a x \Leftrightarrow x = a^y$ where $a > 0$, $a \neq 0$

$a^{\log_a M} = M$

$\log_a(MN) = \log_a M + \log_a N$

$\log_a\left(\frac{M}{N}\right) = \log_a M - \log_a N$

$\log_a M^x = x\log_a M$

$\log_a M = \frac{\log_b M}{\log_b a} = \frac{\log M}{\log a} = \frac{\ln M}{\ln a}$

SPECIAL PRODUCTS

$x^2 - y^2 = (x + y)(x - y)$

$x^3 \pm y^3 = (x \pm y)(x^2 \mp xy + y^2)$

BINOMIAL THEOREM

$(x \pm y)^2 = x^2 \pm 2xy + y^2$

$(x \pm y)^3 = x^3 \pm 3x^2y + 3xy^2 \pm y^3$

$(x + y)^n = x^n + nx^{n-1}y + \frac{n(n-1)}{2}x^{n-2}y^2 + \ldots + \binom{n}{k}x^{n-k} + \ldots + nxy^{n-1} + y^n$

where $\binom{n}{k} = \frac{n(n-1)\ldots(n-k+1)}{1 \cdot 2 \cdot 3 \cdot \ldots \cdot k}$

PASCAL's TRIANGLE OF NUMBERS

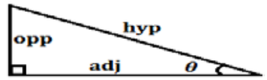
1 1
1 2 1
1 3 3 1
1 4 6 4 1
1 5 10 10 5 1
1 6 15 20 15 6 1
1 7 21 35 35 21 7 1
1 8 28 56 70 56 28 8 1
1 9 36 84 126 126 84 36 9 1
1 10 45 120 210 252 210 120 45 10 1
1 11 55 165 330 462 462 330 165 55 11 1

PYTHAGOREAN THEOREM

$leg^2 + leg^2 = hypotenuse^2$

DISTANCE FORMULA

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$



$\sin\theta = \frac{opp}{hyp}$

$\cos\theta = \frac{adj}{hyp}$

$\tan\theta = \frac{opp}{adj}$

$\cot\theta = \frac{adj}{opp}$

$\csc\theta = \frac{hyp}{opp}$

$\sec\theta = \frac{hyp}{adj}$

$\tan\theta = \frac{\sin\theta}{\cos\theta}$

$\cot\theta = \frac{\cos\theta}{\sin\theta}$

$\sin\theta = \frac{1}{\csc\theta}$

$\cos\theta = \frac{1}{\sec\theta}$

$\csc\theta = \frac{1}{\sin\theta}$

$\sec\theta = \frac{1}{\cos\theta}$

$\tan\theta = \frac{1}{\cot\theta}$

$\cot\theta = \frac{1}{\tan\theta}$

$\sin^2\theta + \cos^2\theta = 1$

$\tan^2\theta + 1 = \sec^2\theta$

$\cot^2\theta + 1 = \csc^2\theta$

$\sin^2\theta = \frac{1 - \cos 2\theta}{2}$

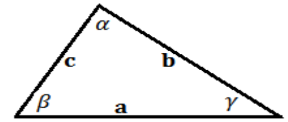
$\cos^2\theta = \frac{1 + \cos 2\theta}{2}$

$\tan^2\theta = \frac{1 - \cos 2\theta}{1 + \cos 2\theta}$

$\sin(2\theta) = 2\sin\theta\cos\theta$

$\cos(2\theta) = \cos^2\theta - \sin^2\theta = 2\cos^2\theta - 1 = 1 - 2\sin^2\theta$

$\tan(2\theta) = \frac{2\tan\theta}{1 - \tan^2\theta}$



Law of Sines

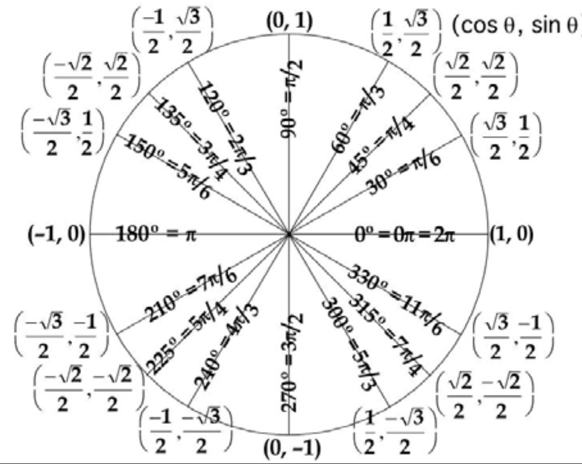
$\frac{\sin\alpha}{a} = \frac{\sin\beta}{b} = \frac{\sin\gamma}{c}$

Law of Cosines

$a^2 = b^2 + c^2 - 2bc\cos\alpha$

$b^2 = a^2 + c^2 - 2ac\cos\beta$

$c^2 = a^2 + b^2 - 2ab\cos\gamma$



Sum and Difference

$\sin(\alpha \pm \beta) = \sin\alpha\cos\beta \pm \cos\alpha\sin\beta$

$\cos(\alpha \pm \beta) = \cos\alpha\cos\beta \mp \sin\alpha\sin\beta$

$\tan(\alpha \pm \beta) = \frac{\tan\alpha \pm \tan\beta}{1 \mp \tan\alpha\tan\beta}$

Sum to Product

$\sin\alpha + \sin\beta = 2\sin\left(\frac{\alpha+\beta}{2}\right)\cos\left(\frac{\alpha-\beta}{2}\right)$

$\sin\alpha - \sin\beta = 2\cos\left(\frac{\alpha+\beta}{2}\right)\sin\left(\frac{\alpha-\beta}{2}\right)$

$\cos\alpha + \cos\beta = 2\cos\left(\frac{\alpha+\beta}{2}\right)\cos\left(\frac{\alpha-\beta}{2}\right)$

$\cos\alpha - \cos\beta = -2\sin\left(\frac{\alpha+\beta}{2}\right)\sin\left(\frac{\alpha-\beta}{2}\right)$

Product to Sum

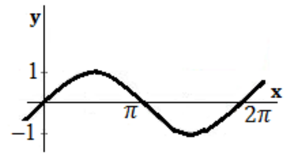
$\sin\alpha\sin\beta = \frac{\cos(\alpha-\beta) - \cos(\alpha+\beta)}{2}$

$\cos\alpha\cos\beta = \frac{\cos(\alpha-\beta) + \cos(\alpha+\beta)}{2}$

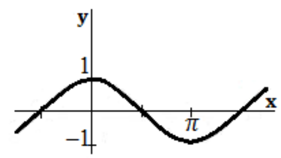
$\sin\alpha\cos\beta = \frac{\sin(\alpha+\beta) + \sin(\alpha-\beta)}{2}$

$\cos\alpha\sin\beta = \frac{\sin(\alpha+\beta) - \sin(\alpha-\beta)}{2}$

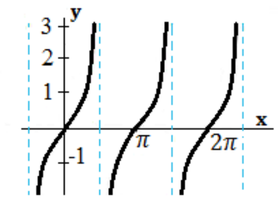
GRAPHS OF THE SIX TRIGONOMETRIC FUNCTIONS



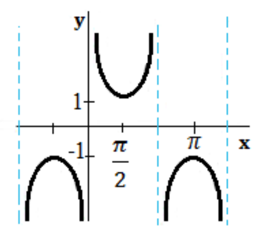
$y = \sin x$
Domain: all reals
Range: $[-1, 1]$
Period: 2π, odd



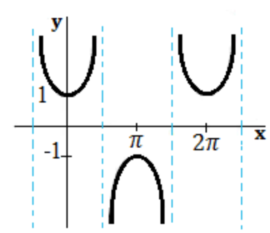
$y = \cos x$
Domain: all reals
Range: $[-1, 1]$
Period: 2π, even



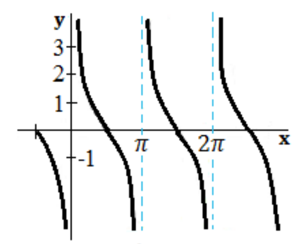
$y = \tan x$
Domain: all reals, $x \neq \frac{\pi}{2} + k\pi$
Range: all reals
Period: π, odd



$y = \csc x$
Domain: all reals, $x \neq n\pi$
Range: $(-\infty, -1]$ and $[1, \infty)$
Period: 2π, odd



$y = \sec x$
Domain: all reals, $x \neq \frac{\pi}{2} + k\pi$
Range: $(-\infty, -1]$ and $[1, \infty)$
Period: 2π, even



$y = \cot x$
Domain: all reals, $x \neq n\pi$
Range: all reals
Period: π, odd

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OPERATIONAL HOURS
Monday thru Thursday 9:00AM – 7:50PM
Friday 10:00AM – 12:50PM
Saturday 12:00PM – 4:00PM

"Who has not been amazed to learn that the function $y = e^x$, like a phoenix rising from its own ashes, is its own derivative?" Francois le Lionnais

DERIVATIVES

Definition:

Derivative: $f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$ if this limit exists.

Applications: If $y = f(x)$ then,

- $m = f'(a)$ is the slope of the tangent line to y=f(x) at x=a and the equation of the tangent line at $x = a$ is given by $y = f(a) + f'(a)(x-a)$.
- $f'(a)$ is the instantaneous rate of change of $f(x)$ at $x = a$.
- If $f(x)$ is the position of an object at time x, then $f'(a)$ is the velocity of the object at $x = a$

Critical points:

$x = c$ is the critical point of $f(x) = c$ provided either **1.** $f'(c) = 0$ or **2.** $f'(c)$ does not exist.

Increasing/Decreasing

- If $f'(x) > 0$ for all x in an interval I, then f(x) is increasing on the interval I.
- If $f'(x) < 0$ for all x in an interval I, then f(x) is decreasing on the interval I.
- If $f'(x) = 0$ for all x in an interval I, then f(x) is constant on the interval I.

Concavity

- If $f''(x) > 0$ for all x in an interval I, then f(x) is concave up on the interval I.
- If $f''(x) < 0$ for all x in an interval I, then f(x) is concave down on the interval I.

Inflection Points

$x = c$ is an inflection point of f(x) if the concavity changes at $x = c$.

COMMON DERIVATIVES

1) $c' = 0$
2) $[f(x) + g(x)]' = f'(x) + g'(x)$
3) $[f(x)g(x)]' = f(x)g'(x) + f'(x)g(x)$
4) $[f(g(x))]' = f'(g(x))g'(x)$
5) $[cf(x)]' = cf'(x)$
6) $[f(x) - g(x)]' = f'(x) - g'(x)$
7) $\left[\frac{f(x)}{g(x)}\right]' = \frac{g(x)f'(x) - f(x)g'(x)}{[g(x)]^2}$
8) $(x^n)' = nx^{n-1}$
9) $[e^x]' = e^x$
10) $[a^x]' = a^x \ln a$
11) $[\ln|x|]' = \frac{1}{x}$
12) $[\log_a x]' = \frac{1}{x\ln a}$
13) $(\sin x)' = \cos x$
14) $(\cos x)' = -\sin x$
15) $(\tan x)' = \sec^2 x$
16) $(\cot x)' = -\csc^2 x$
17) $(\sec x)' = \sec x \tan x$
18) $(\csc x)' = -\csc x \cot x$
19) $(\sin^{-1} x)' = \frac{1}{\sqrt{1-x^2}}$
20) $(\cos^{-1} x)' = -\frac{1}{\sqrt{1-x^2}}$
21) $(\tan^{-1} x)' = \frac{1}{1+x^2}$
22) $(\cot^{-1} x)' = -\frac{1}{1+x^2}$
23) $(\sec^{-1} x)' = \frac{1}{|x|\sqrt{x^2-1}}$
24) $(\csc^{-1} x)' = -\frac{1}{|x|\sqrt{x^2-1}}$
25) $(\sinh x)' = \cosh x$
26) $(\cosh x)' = \sinh x$
27) $(\tanh x)' = \operatorname{sech}^2 x$
28) $(\coth x)' = -\operatorname{csch}^2 x$
29) $(\operatorname{sech} x)' = -\operatorname{sech} x \tanh x$
30) $(\operatorname{csc} x)' = -\operatorname{csch} x \coth x$
31) $(\sinh^{-1} x)' = \frac{1}{\sqrt{1+x^2}}$
32) $(\cosh^{-1} x)' = \frac{1}{\sqrt{x^2-1}}$
33) $(\tanh^{-1} x)' = \frac{1}{1-x^2}$
34) $(\coth^{-1} x)' = \frac{1}{1-x^2}$
35) $(\operatorname{sech}^{-1} x)' = -\frac{1}{|x|\sqrt{1-x^2}}$
36) $(\operatorname{csch}^{-1} x)' = -\frac{1}{|x|\sqrt{x^2+1}}$

INTEGRATION

Definition: Suppose $f(x)$ is continuous on $[a,b]$. Divide $[a,b]$ into n subintervals of width Δx and choose x_i^* from each interval. Then

$$\int_a^b f(x)dx = \lim_{n\to\infty} \sum_{i=1}^{\infty} f(x_i^*)\Delta x \quad \text{where } \Delta x = \frac{(b-a)}{n}$$

Fundamental Theorem of Calculus: Suppose $f(x)$ is continuous on $[a,b]$, then

Part I: $g(x) = \int_a^x f(t)dt$ is also continuous on $[a,b]$ and $g'(x) = \frac{d}{dx}\int_a^x f(t)dt = f(x)$ where $a \le x \le b$.

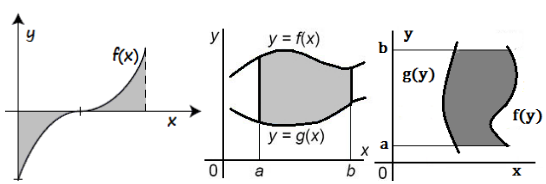
Part II: $\frac{d}{dx}\int_a^b f(x)dx = F(b) - F(a)$ where $F(x)$ is any anti-derivative of $f(x)$, i.e, a function such that $F' = f$.

Applications:

Area: $A = \int_a^b f(x)dx$

Area between Curves:

- $y = f(x)$; $A = \int_a^b (upper - lower\ funtion)dx$
- $x = f(y)$; $A = \int_a^b (right - left\ funtion)\,dy$



Volumes: $V = \int_a^b Area(x)\,dx$

Volume of Revolution

Rings $V = \int_a^b 2\pi(outer\ r^2 - inner\ r^2)$

Cylinders $V = \int_a^b circumference \cdot height \cdot thickness$

Work: If a force of $F(x)$ moves an object in $a \le x \le b$, then the work done is $W = \int_a^b F(x)dx$

Average Function Value: The average value of $f(x)$ on $a \le x \le b$ is $f_{average} = \frac{1}{b-a}\int_a^b f(x)dx$

INTEGRALS

1) $\int u^n du = \frac{u^{n+1}}{n+1} + c,\ n \neq -1$
2) $\int \frac{du}{u} = \ln|u| + c$
3) $\int e^u du = e^u + c$
4) $\int a^u du = \frac{a^u}{\ln a} + c$
5) $\int \ln u\, du = u\ln u - u + c$
6) $\int \frac{1}{u\ln u} du = \ln|\ln u| + c$
7) $\int \sin u\, du = -\cos u + c$
8) $\int \cos u\, du = \sin u + c$
9) $\int \tan u\, du = \ln|\sec u| + c$
10) $\int \cot u\, du = \ln|\sin u| + c$
11) $\int \sec u\, du = \ln|\sec u + \tan u| + c$
12) $\int \csc u\, du = \ln|\csc u - \cot u| + c$
13) $\int \sec^2 u\, du = \tan u + c$
14) $\int \csc^2 u\, du = -\cot u + c$
15) $\int \sec u \tan u\, du = \sec u + c$
16) $\int \csc u \cot u\, du = -\csc u + c$
17) $\int \frac{du}{\sqrt{a^2-u^2}} = \sin^{-1}\frac{u}{a} + c,\ a > 0$
18) $\int \frac{du}{a^2+u^2} = \frac{1}{a}\tan^{-1}\frac{u}{a} + c$
19) $\int \frac{du}{u\sqrt{u^2-a^2}} = \frac{1}{a}\sec^{-1}\frac{u}{a} + c$
20) $\int \frac{du}{a^2-u^2} = \frac{1}{2a}\ln\left|\frac{u+a}{u-a}\right| + c$
21) $\int \frac{du}{u^2-a^2} = \frac{1}{2a}\ln\left|\frac{u-a}{u+a}\right| + c$
22) $\int \sin^{-1} u\, du = u\sin^{-1} u + \sqrt{1-u^2+c}$
23) $\int \cos^{-1} u\, du = u\cos^{-1} u + \sqrt{1-u^2+c}$
24) $\int \tan^{-1} u\, du = u\tan^{-1} u - \frac{1}{2}\ln(1+u^2) + c$
25) $\int \sinh u\, du = \cosh u + c$
26) $\int \cosh u\, du = \sinh u + c$
27) $\int \tanh u\, du = \ln(\cosh u) + c$
28) $\int \coth u\, du = \ln|\sinh u| + c$
29) $\int \operatorname{sech} u\, du = \tan^{-1}|\sinh u| + c$
30) $\int \operatorname{csch} u\, du = \ln\left|\tanh\frac{1}{2}u\right| + c$
31) $\int \operatorname{sech}^2 u\, du = \tanh u + c$
32) $\int \operatorname{csch}^2 u\, du = -\coth u + c$
33) $\int \operatorname{sech} u \tanh u\, du = -\operatorname{sech} u + c$
34) $\int \operatorname{csch} u \coth u\, du = -\operatorname{csch} u + c$
35) $\int u\,dv = uv - \int v\,du$

"Success is the sum of small efforts, repeated day in and day out." Robert Collier

"Go down deep enough into anything and you will find mathematics." Dean Schlicter