1) Country workshop manufactures both finished and unfinished furniture for the home. The estimated quantities demanded each week of its rolltop desks in the finished and unfinished versions are x and y units when the corresponding unit prices are:

\[
p = 200 - \frac{1}{5}x - \frac{1}{10}y \quad \text{dollars respectively.}
\]

\[
q = 160 - \frac{1}{10}x - \frac{1}{4}y
\]

a) Find the Revenue function \( R(x, y) \)

b) Compute \( R(100, 60) \) and Interpret

2) Find the first order partial derivatives of each:

a) \( f(x, y) = x^2 - xy^2 + y^3 \)

b) \( f(x, y) = x \ln y + y \ln x \)

c) \( f(x, y) = 3x^2 - 4xy^2 + 6y^3 + xe^{3y} \)

d) \( f(x, y) = (x^2 - xy + y^2)^5 \)

3) Find the second order partial derivatives of \( f(x, y) = x^3 - 3x^2y + 3xy^2 + y^2 \)

4) The total weekly revenue of the Country workshop associated with manufacturing and selling their rolltop desks is given by:

\[
R(x, y) = -0.2x^2 - 0.25y^2 - 0.2xy + 200x + 160y
\]

where \( x \) denotes the number of finished units and \( y \) the number of unfinished units manufactured and sold each week. Compute and Interpret \( R_x(x, y) \) and \( R_y(x, y) \) when \( x = 300 \) and \( y = 250 \).

5) Find any relative extrema or saddle points for:

\( f(x, y) = 4y^3 + x^2 - 12y^2 - 36y + 2 \)

6) The total weekly revenue in dollars that Acrosonic realizes in producing and selling its bookshelf speakers is given by:

\[
R(x, y) = -\frac{1}{4}x^2 - \frac{3}{8}y^2 - \frac{1}{4}xy + 300x + 240y
\]

where \( x \) denotes the number of fully assembled units and \( y \) the number of unassembled kits produced and sold each week. The total weekly cost for the production of these speakers is

\[
C(x, y) = 180x + 140y + 5000
\]

dollars. Determine how many of each type of speaker units should be produced and sold each week to maximize profit. What is the maximum weekly profit?

7) Evaluate each Integral:

a) \( \int_{0}^{4} 3xy^2 \, dy \)

b) \( \int_{0}^{2} (3x + 5y) \, dx \)

c) \( \int_{1}^{4} (x + 2y) \, dx \)

d) \( \int_{0}^{2} \int_{x}^{2} (x^2 + y^2) \, dy \, dx \)
Answers:

1) a. see problem 4  b. the revenue is $25500 when selling 100 finished and 60 unfinished desks

2) a. \( f_x = 2x - y^2 \) and \( f_y = -2xy + 3y^2 \)  b. \( f_x = \ln(y) + \frac{y}{x} \) and \( f_y = \frac{y}{x} + \ln x \)

c. \( f_x = 6x - 4y^2 + e^{3y} \) and \( f_y = -8xy + 18y^2 + 3xe^{3y} \)  d. \( f_x = 5(2x - y)(x^2 - xy + y^2)^4 \) and \( f_y = 5(2y - x)(x^2 - xy + y^2)^4 \)

3) \( f_{xx} = 6x - 6y \)  \( f_{yy} = 6x + 2 \)  \( f_{xy} = f_{yx} = -6x + 6y \)

4) The weekly revenue increases by $30/unit for each additional finished desk produced (beyond 300) when the level of production of unfinished desks remains fixed at 250. Also, the revenue decreases by $25/unit when each additional unfinished desk (beyond 250) is produced and the level of production of finished desks remains fixed at 300.

5) Saddle point at (0, -1), Relative Minimum occurs at (0,3) this relative minimum is -106

6) If they produce 208 assembled units and 64 unassembled kits the weekly profit will be maximized at $10,680.

7) a. 64x  b. 6 + 10y  c. 33/2  d. 40/3