11.7 Strategies for Testing Series

Let $\sum_{n=1}^{\infty} a_n$ be a series.

1. If $a_n = \frac{1}{n^p}$, then $\sum_{n=1}^{\infty} a_n$ is a **P-Series**. $\sum_{n=1}^{\infty} a_n$ **converges** for $p > 1$ and **diverges** for $p \le 1$.

2. If $a_n = a_1 r^{n-1}$, then $\sum_{n=1}^{\infty} a_n$ is a **Geometric Series**.

$\sum_{n=1}^{\infty} a_n$ **converges** to $\frac{a_1}{1-r}$ for $|r| < 1$, and $\sum_{n=1}^{\infty} a_n$ **diverges** for $|r| \ge 1$.

3. If $\sum_{n=1}^{\infty} a_n$ is similar to **G.S** or **P-Series**, then use **C.T.** or **L.C.T**. $\left(a_n \ge 0\, ;\, \forall\, n \ge n_0 \right)$

4. If $\lim_{x \to \infty} a_n \ne 0$ (if it is easy to evaluate), then $\sum_{n=1}^{\infty} a_n$ **diverges** by **D.T**. .

5. Let $b_n > 0$ and $a_n = (-1)^n b_n$ or $a_n = (-1)^{n+1} b_n$; $\forall n \ge n_0$, then $\sum_{n=1}^{\infty} a_n$ is an A.S.,

and if $i)\ b_{n+1} \le b_n$ and $ii)\ \lim_{x \to \infty} b_n = 0$, then $\sum_{n=1}^{\infty} a_n$ **converges** by **A.S.T.** .

6. Let $\lim_{x \to \infty} \left| \frac{a_{n+1}}{a_n} \right| = L$, then $\sum_{n=1}^{\infty} a_n$ **converges** for $L < 1$ and **diverges** for $L > 1$ by **ratio** test.

7. Let $\lim_{x \to \infty} \sqrt[n]{|a_n|} = L$, then $\sum_{n=1}^{\infty} a_n$ **converges** for $L < 1$ and **diverges** for $L > 1$ by **root** test.

8. Let $a_n = f(n)$, $f(x) > 0$, continuous and decreasing $\forall\, x \ge x_0$.

If $\int_{x_0}^{\infty} f(x)\, dx$ converges, then $\sum_{n=1}^{\infty} a_n$ **converges** by **I.T**., otherwise it **diverges**.