# EXPERIMENT 12: SIMPLE HARMONIC MOTION

Simple harmonic motion (SHM) appears in many areas of physics and life. It is important because many types of harmonic motion (which means any repeated motion) can be understood as combinations of simple harmonic motion. In this experiment a vertically hanging spring will provide us with an example of SHM. We will determine the force constant of the spring by two methods.

## BACKGROUND

You should review the sections in the book relating to SHM and springs, sections 15-2 through 15-4. The basic equations are $F = - kx$ and $U = \frac{1}{2}kx^2$, as well as $x = x_m\cos(\omega t + \phi)$. Recall also the relation between velocity and position ($v = dx/dt$). We will use a direct method, involving the definition of Hooke's law and a less direct method, involving the period of a simple harmonic oscillator to calculate k.

## ADVANCED STUDY ASSIGNMENT

1. Use the formula $T = 2\pi\sqrt{(m/k)}$ and show how k can be determined from a combination of constants and the slope of a graph of m vs $T^2$.
2. How are the maximum values of x, v and a related in a case of SHM? Use the basic equation of SHM to answer this question.

## PROCEDURE A

1. Hang the spring from a cross bar attached to a ring stand near the end of a table. The ring stand should be clamped to the table with a C-clamp. Place a 50-g hanger on the lower end of the spring. This will stretch the spring slightly, but we will not include its weight in any calculations. Essentially the hanger provides another constant force, like gravity, which we can show (as in Question 1) does not affect the SHM.
2. Next, hold a 2-meter stick, with the 0 reading end on the ground, next to the spring. Use a caliper jaw on the meter stick to point to the position of the bottom of the 50-g hanger. Record the initial position of the caliper jaw, with only the hanger on the spring. Call this position $x_o$ and record it on Data Table A.
3. Now add successive multiples of 50 g, up to 400 g in addition to the hanger, and record the position of the bottom of the hanger each time, in Data Table A.
4. Complete the rest of the data table by calculations, $F = mg$ and $\Delta x_i = x_i - x_o$. You may either do the calculations by hand and fill in on your data table, or you may call up the data table on the Experiment 12 Data Table file, enter the numbers in the data table on the computer, and write a formula on the computer to do the calculations.
5. Next, draw a graph of F vs $\Delta x$. Again, you may either do this by hand on graph paper, or do the graph on an Excel spreadsheet on the computer. If you use the computer for either job, print a copy of the data table and/or graph, including the slope of the graph. Recall that instructions for graphing with the Excel are saved either on the computer or on a floppy available from the instructor.

**PROCEDURE B**

6. Now use masses of 50, 100, 200, 300 and 400 g (mass excluding hanger) and determine the period of oscillation. This is conveniently done by putting a motion sensor on the floor under the spring, and using the computer to display position, velocity and acceleration during the SHM.
7. Select Motion Sensor under Experiment Setup. Next, drag each of the three variables in the Data Window (position, velocity, acceleration) onto Graph (to display three graphs). The second and third should be dragged onto Graph 1.
8. Put 50 g on the hanger and start the spring oscillating. Click "Start" on the Data Studio Experiment window. Stop after 10 or more complete oscillations show on the graphs. Select each graph in turn by clicking on its area, so the title is highlighted in yellow. Then click on the autofit button (top left of the graph window menu bar).
9. Select the smart tool button function (a sketch of an x-y coordinate axis five buttons right of the autofit button). Now move the cursor to any peak on one of the graphs, and record the time displayed for that peak. Move the cursor to a peak 10 or so complete oscillations away and again record the time, both times to be recorded on Data Table B. Calculate $\Delta t$ and record N, the number of complete oscillations between the two times. With the last two numbers, calculate T, the period of the oscillation. All calculations may be done by hand, or by using an Excel spreadsheet, and entering the data and formulas on Experiment 12 Data Table B in the file.
10. Complete the data table for the other masses shown, up to 400 g. Next calculate $T^2$ and draw a graph of m vs $T^2$ (either by hand or by computer). Note the next instruction while you are doing the 200 g mass.
11. For the 200 g mass, record the maximum and minimum values of a, x and v in the space at the end of Data Table B. Then calculate the amplitudes for each value (for x, this will be one-half the difference of the maximum and minimum recorded; for v and a the amplitude will be the average of the absolute values of the recorded maximum and minimum).
12. Now calculate k from the slope of the graph and the results of the Advanced Study Assignment.
13. Calculate the percent difference between the two values of k.

**QUESTIONS**

1. The requirement of SHM is a linear restoring force. In this experiment, there are actually two forces acting, the constant force of gravity, and the spring force. Show that the combined force is a linear restoring force, so that this is actually an example of SHM (although the graphs you observed also show this result). Hint: Draw a FBD and consider net forces at equilibrium and with a small displacement from equilibrium.
2. Which value of k, from Data Table A or Data Table B, do you think is more likely closer to the actual value? How would you determine which is more accurate? Can you think of a third experiment or a calculation which would help to decide? Hint: (Look at ASA 2.)
3. Using the maximum value of $x = x_m$ from DT B, m = 200 g, calculate the maximum values of v and a, using $\omega$ (= $2 \pi f$) and the basic equation of SHM and its two derivatives. Compare these calculated values for $a_m$ and $v_m$ with the values in the bottom of Data Table B.

**DATA TABLE A**

| m(g) | x(cm) | Δx (cm) | F (dynes) |
|---|---|---|---|
| hangar | | 0 | |
| 50 | | | |
| 100 | | | |
| 150 | | | |
| 200 | | | |
| 250 | | | |
| 300 | | | |
| 350 | | | |
| 400 | | | |

Calculated value of k from graph: ________________

**DATA TABLE B**

| $t_i$(s) | $t_f$(s) | Δt(s) | N | T(s) | m(g) | $T^2$($s^2$) |
|---|---|---|---|---|---|---|
| | | | | | 50 | |
| | | | | | 100 | |
| | | | | | 200 | |
| | | | | | 300 | |
| | | | | | 400 | |

Calculated value of k from graph: ____________

From m = 200g record:

Max a _____________ Min a ____________ $a_m$ ________________

Max v ______________ Min v ____________ $v_m$ ________________

Max x ______________ Min x ____________ $x_m$ ________________

%diff (v) ________________

%diff (a) ________________